1. You probably figured this one out :)

2. The switched dc-dc step-down converter shown in Figure 1 controls a dc machine with an armature inductance $L_\mathrm{a} = 0.2\,\mathrm{mH}$. The armature resistance can be neglected. The armature current i_o is 5 A. The switching frequency $f_\mathrm{sw} = 30\,\mathrm{kHz}$ and the duty cycle, $D = 0.8$. Consider all the components to be ideal.

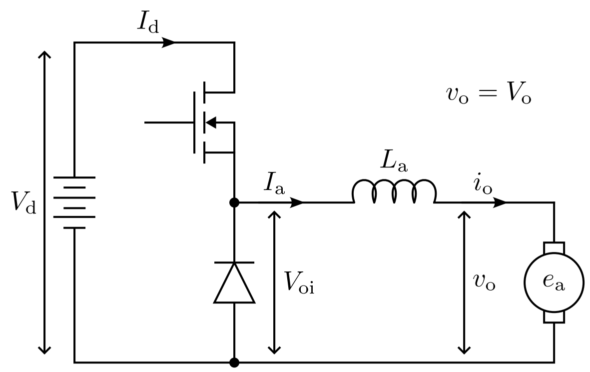


Figure 1: Step-down DC-DC converter.

(a) **The output voltage, $V_\mathrm{o} = 200\,\mathrm{V}$. Calculate the input voltage, V_d.**
The duty cycle is given by

$$D = \frac{V_\mathrm{o}}{V_\mathrm{d}}$$

The input voltage is

$$V_\mathrm{d} = \frac{V_\mathrm{o}}{D} = \frac{200}{0.8} = 250\,\mathrm{V}.$$

(b) **Find the ripple in the armature current.**
The peak-to-peak armature (iunductor) current ripple (ΔI_a) is

$$\Delta I_\mathrm{a} = \frac{D}{L\,f_\mathrm{sw}}\,(V_\mathrm{d} - V_\mathrm{o}) = \frac{0.8}{200 \times 10^{-6} \times 30 \times 10^{3}}\,(250 - 200) = 6.67\,\mathrm{A}$$

(c) **Calculate the maximum and the minimum value of the armature current.**
The average armature current $I_\mathrm{a} = 5\,\mathrm{A}$. The peak armature current ripple δI_a is

$$\delta I_\mathrm{a} = \frac{\Delta I_\mathrm{a}}{2} = 3.33\,A$$

The maximum and minimum values of the armature current ($I_\mathrm{a}^\mathrm{max}$ and $I_\mathrm{a}^\mathrm{min}$, respectively) are

$$I_\mathrm{a}^\mathrm{max} = I_\mathrm{a} + \delta I_\mathrm{a} = 8.33\,\mathrm{A}$$
$$I_\mathrm{a}^\mathrm{min} = I_\mathrm{a} - \delta I_\mathrm{a} = 1.67\,\mathrm{A}$$

(d) **The load on the machine is reduced. Calculate I_a when the converter is on the boundary between continuous and discontinuous mode.**
The peak-to-peak armature (iunductor) current ripple (ΔI_a) is

$$\Delta I_\mathrm{a} = \frac{D}{L\,f_\mathrm{sw}}\,(V_\mathrm{d} - V_\mathrm{o}) = \frac{D}{L\,f_\mathrm{sw}}\,(V_\mathrm{d} - D\,V_\mathrm{d}) = \frac{D\,(1-D)}{L\,f_\mathrm{sw}}\,V_\mathrm{d}.$$

From the above equation, it is clear that ΔI_a is maximum when $D = 0.5$. Therefore, The armature current ripple when the converter is on the boundary between continuous and discontinuous mode ($\Delta I_\mathrm{a}^\mathrm{B}$) is

$$\Delta I_\mathrm{a}^\mathrm{B} = \frac{V_\mathrm{d}}{4\,L\,f_\mathrm{sw}} = \frac{250}{4 \times 0.2 \times 10^{-3} \times 30 \times 10^{3}} = 10.42\,\mathrm{A}.$$

The average armature current (I_a^B) is

$$I_\mathrm{a}^\mathrm{B} = \frac{I_\mathrm{a}^\mathrm{B}}{2} = \frac{10.42}{2} = 5.21\,\mathrm{A}.$$

(e) **The load on the DC machine gives $I_\mathrm{a}' = 2$ A. Is the converter in discontinuous mode? Note: The duty cycle of the converter is changed.**

If the armature current is 5 A (I_a) at 50% duty-cycle (D), then the armature current (I_a') is 2 A, the duty-cycle (D') is

$$D' = \frac{I_\mathrm{a}' D}{I_\mathrm{a}} = \frac{2 \times 0.8}{5} = 0.32.$$

The peak-to-peak armature (iunductor) current ripple ($\Delta I_\mathrm{a}'$) is

$$\Delta I_\mathrm{a}' = \frac{D'\,(1 - D')}{L\,f_\mathrm{sw}}\,V_\mathrm{d} = \frac{0.32\,(1 - 0.32)}{200 \times 10^{-6} \times 30 \times 10^{3}} \times 250 = 9.07\,\mathrm{A}$$

The average armature current $I_\mathrm{a}' = 2$ A. The peak armature current ripple $\delta I_\mathrm{a}'$ is

$$\delta I_\mathrm{a}' = \frac{\Delta I_\mathrm{a}'}{2} = 4.53\,A$$

The maximum and minimum values of the armature current ($I_\mathrm{a}^{'\mathrm{max}}$ and $I_\mathrm{a}^{'\mathrm{min}}$, respectively) are

$$I_\mathrm{a}^{'\mathrm{max}} = I_\mathrm{a}' + \delta I_\mathrm{a}' = 6.53\,\mathrm{A}$$
$$I_\mathrm{a}^{'\mathrm{min}} = I_\mathrm{a}' - \delta I_\mathrm{a}' = -2.53\,\mathrm{A}$$

Since $I_\mathrm{a}^{'\mathrm{min}} < 0$, the converter is in discontinuous mode.

3. An n-channel power MOSFET, a VMO 400-02F made by IXYS, is to be used in a converter (datasheet is attached). The MOSFET is to conduct a continuous current of 300 A when on and the switching frequency is 10 kHz with a 50% duty cycle. The input voltage is 100 V. The internal junction temperature is not to exceed 100°C and the maximum ambient temperature is 35°C.

   (a) **Calculate the total MOSFET power losses (assume the same time for voltage and current transients, i.e., $t_{\mathbf{ri}} = t_{\mathbf{fv}} = t_{\mathbf{r}}$ and $t_{\mathbf{fi}} = t_{\mathbf{rv}} = t_{\mathbf{f}}$).**

   From the MOSFET datasheets, the on-state resistance of the MOSFET ($r_{ds(\text{on})}$) and the switching transient times are

$$r_{ds(\text{on})} = 4.2\,\text{m}\Omega \qquad t_{ri} = t_{fv} = 500\,\text{ns}, \qquad t_{rv} = t_{fi} = 350\,\text{ns}.$$

   The conduction losses of the MOSFET (P_c^l) is

$$P_c^l = D\,I_{\text{v(max)}}^2\,r_{ds(\text{on})} = 0.5 \times 300^2 \times 4.2 \times 10^{-3} = 189\,\text{W}.$$

   The switching losses of the MOSFET (P_s^l) is

$$\begin{aligned} P_s^l &= \frac{1}{2}\,V_\text{d}\,I_\text{v}\,t_\text{sw}\,f_\text{sw} = \frac{1}{2}\,V_\text{d}\,I_\text{v}\,\left(t_{ri} + t_{fv} + t_{rv} + t_{fi}\right)\,f_\text{sw} \\ &= \frac{1}{2} \times 100 \times 300 \times \left(2 \times 500 \times 10^{-9} + 2 \times 350 \times 10^{-9}\right) \times 10 \times 10^3 = 255\,\text{W}. \end{aligned}$$

   The total losses in the MOSET (P^l) is

$$P^l = P_c^l + P_s^l = 189 + 255 = 444\,\text{W}.$$

   (b) **Specify the thermal resistance of the required heat sink (Assume that the MOSTFET has no heat transfer paste).**

   The thermal resistance of the required heat sink is calculated as follows:

$$\Delta T_\text{ja} = P^l\,\left(R_{\theta\text{jc}} + R_{\theta\text{ca}}\right) \qquad \Longrightarrow\; R_{\theta\text{ca}} = \frac{\Delta T_\text{ja}}{P^l} - R_{\theta\text{jc}}$$

$$R_{\theta\text{ca}} = \frac{100 - 35}{444} - 0.051 = 0.0954\,\text{K/W}.$$

   (c) **Determine the peak MOSFET drain-to-source voltage during the switching transient if the blocking voltage (MOSFET drain-to-source voltage when it is completely turned off) is 100 V and the parasitic inductance near the MOSFET drain terminal is 100 nH.**

   The voltage across the MOSFET during the turn-off transient is

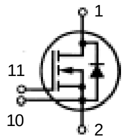
$$V_\text{ds}^\text{max} = V_\text{d} - L\,\frac{dV_\text{ds}}{dt} = 100 - 100 \times 10^{-9} \times \frac{0 - 300}{350 \times 10^{-9}} = 185.71\,\text{V}.$$

MegaMOS™FET Module

VMO 400-02F

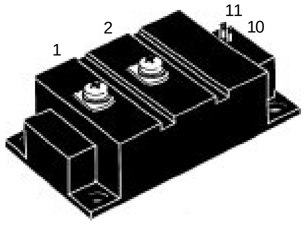
V_{DSS} = 200 V
I_{D25} = 418 A
$R_{DS(on)}$ = 4.2 mΩ

N-Channel Enhancement Mode



Symbol	Test Conditions		Maximum Ratings	
V_{DSS}	T_J = 25°C to 150°C		200	V
V_{DGR}	T_J = 25°C to 150°C; R_{GS} = 10 kΩ		200	V
V_{GS}	Continuous		±20	V
V_{GSM}	Transient		±30	V
I_{D25}	T_K = 25°C		418	A
I_{DM}	T_K = 25°C, t_P = 10 µs		1672	A
P_D	T_C = 25°C T_K = 25°C		2450 1640	W W
T_J			-40 ...+150	°C
T_{JM}			150	°C
T_{stg}			-40 ... +125	°C
V_{ISOL}	50/60 Hz I_{ISOL} ≤ 1 mA	t = 1 min t = 1 s	3000 3600	V~ V~
M_d	Mounting torque (M6) Terminal connection torque (M5)		2.25-2.75/20-25 2.5-3.7/22-33	Nm/lb.in. Nm/lb.in.
Weight	typical including screws		250	g

Symbol	Test Conditions	Characteristic Values (T_J = 25°C, unless otherwise specified) min.	typ.	max.	
V_{DSS}	V_{GS} = 0 V, I_D = 12 mA	200			V
$V_{GS(th)}$	V_{DS} = 20 V, I_D = 120 mA	3		6	V
I_{GSS}	V_{GS} = ±20 V DC, V_{DS} = 0			±500	nA
I_{DSS}	V_{DS} = V_{DSS}, V_{GS} = 0 V T_J = 25°C V_{DS} = 0.8 • V_{DSS}, V_{GS} = 0 V T_J = 125°C			2.5 12	mA mA
$R_{DS(on)}$	V_{GS} = 10 V, I_D = 0.5 • I_{D25} Pulse test, t ≤ 300 µs, duty cycle d ≤ 2 %			4.2	mΩ



1 = Drain 2 = Source
10 = Kelvin Source 11 = Gate

Features

- International standard package
- **D**irect **C**opper **B**onded Al_2O_3 ceramic base plate
- Isolation voltage 3600 V~
- Low $R_{DS(on)}$ HDMOS™ process
- Low package inductance for high speed switching
- Kelvin Source contact for easy drive

Applications

- AC motor speed control for electric vehicles
- DC servo and robot drives
- Switched-mode and resonant-mode power supplies
- DC choppers

Advantages

- Easy to mount
- Space and weight savings
- High power density
- Low losses

IXYS reserves the right to change limits, test conditions, and dimensions.

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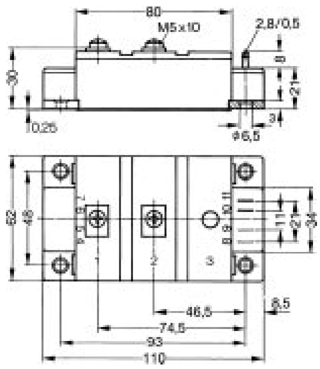
Symbol	Test Conditions	Characteristic Values (T_J = 25°C, unless otherwise specified) min.	typ.	max.	
g_{fs}	V_{DS} = 10 V; I_D = 0.5 • I_{D25} pulsed		380		S
C_{iss}	V_{GS} = 0 V, V_{DS} = 25 V, f = 1 MHz		53		nF
C_{oss}			9.6		nF
C_{rss}			3.4		nF
$t_{d(on)}$	V_{GS} = 10 V, V_{DS} = 0.5 • V_{DSS}, I_D = 0.5 • I_{D25}		210		ns
t_r	R_G = 1 Ω (External)		500		ns
$t_{d(off)}$			900		ns
t_f			350		ns
Q_g	V_{GS} = 10 V, V_{DS} = 0.5 • V_{DSS}, I_D = 0.5 • I_{D25}		2300		nC
Q_{gs}			420		nC
Q_{gd}			1150		nC
R_{thJC}				0.051	K/W
R_{thJK}	with 30 µm heat transfer paste			0.076	K/W

Dimensions in mm (1 mm = 0.0394")



Source-Drain Diode

Symbol	Test Conditions	Characteristic Values (T_J = 25°C, unless otherwise specified) min.	typ.	max.	
I_S	V_{GS} = 0 V			418	A
I_{SM}	Repetitive; pulse width limited by T_{JM}			1672	A
V_{SD}	I_F = I_S; V_{GS} = 0 V, Pulse test, t ≤ 300 µs, duty cycle d ≤ 2 %		0.9	1.2	V
t_{rr}	I_F = I_S, -di/dt = 1200 A/µs, V_{DS} = 100 V		600		ns

IXYS MOSFETS and IGBTs are covered by one or more of the following U.S. patents:
4,835,592 4,881,106 5,017,508 5,049,961 5,187,117 5,486,715
4,850,072 4,931,844 5,034,796 5,063,307 5,237,481 5,381,025

4. The output voltages and current of a singe-phase inverter are shown in the figure. Determine the following:

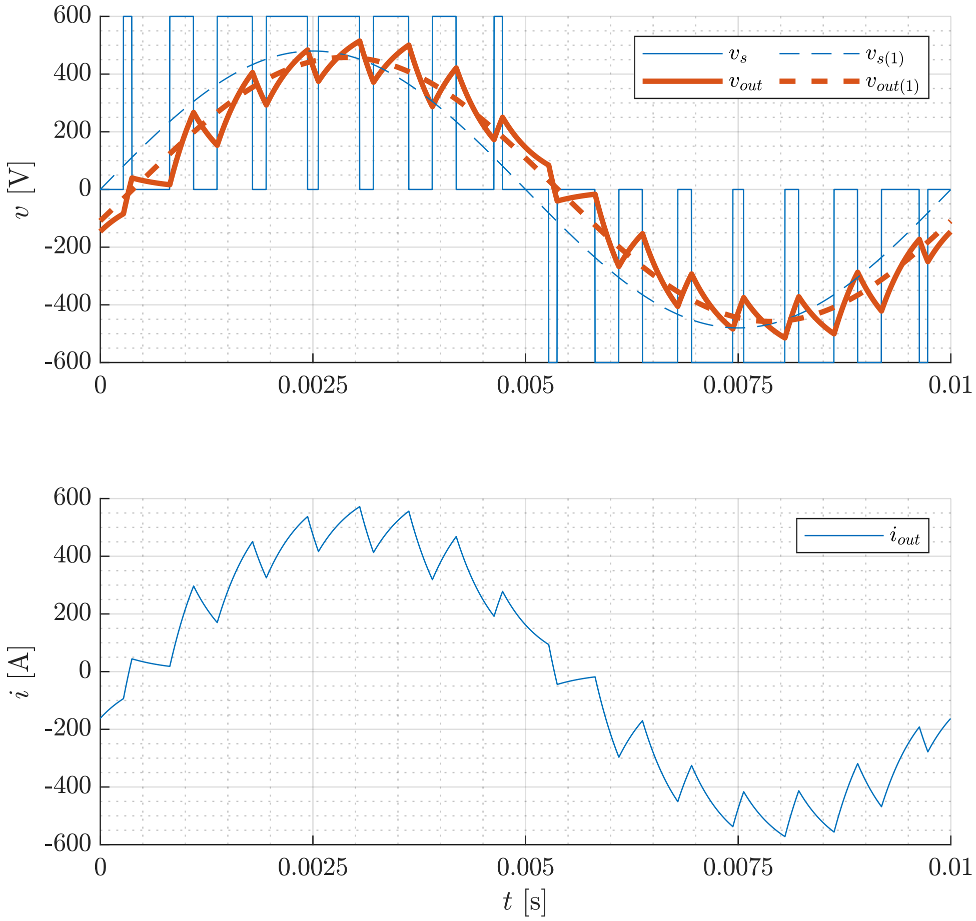


Figure 2: full-bridge inverter output waveforms.

(a) **Type of modulation (unipolar or bipolar).**
Unipolar modulation.

(b) **Switching frequency**
Counting the number of positive/negative pulses (or, rising/falling edges) for v_s in Figure 2, gives $2 \times m_f$ as

$$2\, m_f = 16 \quad \implies m_f = 8.$$

Since m_f is defined as

$$m_f = \frac{f_{sw}}{f_1} \qquad\qquad \implies f_{sw} = m_f\, f_1,$$

where f_1 is the fundamental frequency, and from Figure 2

$$f_1 = \frac{1}{0.01\,\mathrm{s}} = 100\,\mathrm{Hz}.$$

Therefore,

$$f_{sw} = m_f\, f_1 = 800\,\mathrm{Hz}.$$

(c) **Inductance.**

From Figure 2, during the time interval $t \in [0.0025, 0.003]$ s, the $v_s = 600$ V and for simplicity the voltage after the inductor v_{out} is about 450 V. Then the voltage drop across the inductor is

$$V_L = (v_{out}(t) - v_s(t))\Big|_{t\in[0.0025,0.003]} = L\frac{di(t)}{dt}\Big|_{t\in[0.0025,0.003]}$$

$$\Longrightarrow L = \frac{v_{out}(t) - v_s(t)}{\frac{di}{dt}}\Big|_{t\in[0.0025,0.003]}.$$

$dt = 0.003\,\text{s} - 0.0025\,\text{s}$, and from Figure 2, $di = 500\,\text{A} - 400\,\text{A} = 100\,\text{A}$. Therefore the inductance, L is

$$L = \frac{600 - 450}{\frac{100}{0.0005}} = 0.5\,\text{mH}.$$

(d) **Peak fundamental current.**

The peak fundamental output current occurs at $0.0025\,\text{s} \leq t \leq 0.003\,\text{s}$. The average current in this interval is the peak Fundamental output current ($\hat{i}_{out(1)}$)

$$\hat{i}_{out(1)} = \frac{410 + 570}{2} = 490\,\text{A}.$$

(e) **Pole-to-pole DC-link voltage (V_d) and modulation index (m_a).**

In the full-bridge inverter, the pole-to-pole DC-link voltage (V_d) is

$$V_d = \hat{v}_s = 600\,\text{V}.$$

The modulation index (m_a) is

$$m_a = \frac{\hat{v}_{s(1)}}{V_d} = \frac{480}{600} = 0.8.$$

(f) **Active power on the load at the fundamental frequency.**

The active power on the load (P_{out}) at the fundamental frequency is

$$P_{out} = \frac{\hat{v}_{out(1)}\,\hat{i}_{out(1)}}{2} = \frac{455 \times 490}{2} = 111.5\,\text{kW}.$$

(g) **Phase angle of the fundamental current with respect to the inverter side voltage.**

At time $t = 0$ s the reference signal (fundamental converter output voltage) is 0 V thus the converter output voltage (v_s) is also assumed to be 0 V. However, the fundamental load current (or voltage) is 0 A (or 0 V) at $t = 0.0004$ s. The time delay (δt) is

$$\delta t = 0.0004\,\text{s}.$$

If time $t = T = 0.01$ s is 2π, then time $t = \delta t$, i.e., phase angle (ϕ) is

$$\phi = \frac{\delta t}{T}\,2\,\pi = \frac{0.0004}{0.01} \times 2 \times \pi = 14.4^\circ.$$

(h) **Active and reactive power on the converter at the fundamental frequency.**

The active power (P_s) on the converter side is

$$P_s = \frac{\hat{v}_{s(1)}\,\hat{i}_{out(1)}}{2}\cos(\phi) = \frac{480 \times 490}{2}\cos(14.4^\circ) = 113.91\,\text{kW}.$$

The reactive power (Q_s) on the converter side is

$$Q_s = \frac{\hat{v}_{s(1)}\,\hat{i}_{out(1)}}{2}\sin(\phi) = \frac{480 \times 490}{2}\sin(14.4^\circ) = 29.25\,\text{kVar}.$$

5. The problem with ripple in the output current from a single-phase full bridge converter is to be studied. The first harmonic of the output voltage is given by $V_{o(1)}$ at $f_1 = 50\,\text{Hz}$. The load is given in the figure as $L = 10\,\text{mH}$ in series with an ideal voltage source $e_o(t)$. It is assumed that the converter operates in sinusoidal PWM mode, bipolar modulation.

$$e_o(t) = \sqrt{2} \cdot 220 \sin(2\pi f_1 t)$$

Table 1: Generalized harmonics of a half-bridge inverter output voltage for a large m_f.

$h \downarrow \quad m_a \rightarrow$	0.2	0.4	0.6	0.8	1
1 Fundamental	0.2	0.4	0.6	0.8	1
m_f	1.242	1.15	1.006	0.818	0.6023
$m_f \pm 2$	0.061	0.061	0.131	0.22	0.318
$m_f \pm 4$					0.018
$2m_f \pm 1$	0.19	0.326	0.37	0.314	0.181
$2m_f \pm 3$		0.024	0.071	0.139	0.212
$2m_f \pm 5$				0.013	0.033
$3m_f$	0.335	0.123	0.083	0.171	0.133
$3m_f \pm 2$	0.044	0.139	0.203	0.176	0.062
$3m_f \pm 4$		0.012	0.047	0.104	0.157
$3m_f \pm 6$				0.016	0.044
$4m_f \pm 1$	0.163	0.157	0.088	0.105	0.068
$4m_f \pm 3$	0.012	0.070	0.132	0.115	0.009
$4m_f \pm 5$			0.034	0.084	0.119
$4m_f \pm 7$				0.017	0.05

Note: output voltage ($\hat{V}_o$) is $\hat{V}_o = m_a V_d/2$.

Note: Ripple here is referred to as distortion, which is the alteration of the original shape of a signal. Here ripple means the alteration of the waveform from an ideal sinusoidal signal.

(a) **The frequency of the triangular signal is 1050 Hz. Calculate the frequency modulation ratio (or pulse number), m_f.**

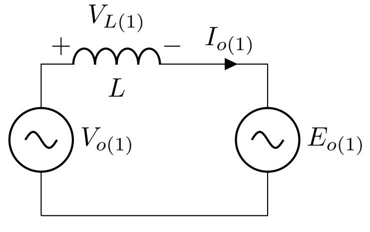
$$m_f = \frac{f_{\text{sw}}}{f_1} = \frac{1050}{50} = 21.$$

(b) **Find the dc-voltage when the converter fundamental RMS output voltage $V_{o(1)}$ is 230 V and modulation index, $m_a = 0.6$.**

$$V_{\text{d}} = \frac{\hat{V}_{o(1)}}{m_a} = \frac{\sqrt{2} \times 230}{0.6} = 542.12\,\text{V}.$$

(c) **Determine the RMS fundamental output current (i.e., current through the inductor).**

The equivalent circuit at the fundamental frequency is given as follows



From the figure,

$$E_{o(1)} = V_{L(1)} + V_{o(1)} \quad \Longrightarrow V_{L(1)} = V_{o(1)} - E_{o(1)} = 230 - 220 = 10\,\text{V}$$

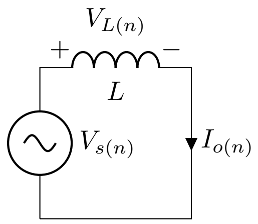
Also,

$$V_{L(1)} = \omega_n \, L \, I_{o(n)} = 2\pi \, n \, f_1 \, L \, I_{o(n)}.$$

Therefore, the average RMS fundamental output current ($I_{o(1)}$), is

$$I_{o(1)} = \frac{V_{L(1)}}{2\pi \, f_1 \, L} = \frac{10}{2 \times \pi \times 50 \times 10 \times 10^{-3}} = 3.18\,\text{A}.$$

(d) **Determine the RMS and frequency of the highest output ripple current component.**
The equivalent circuit at the n^{th} harmonic, where $n \neq 1$ is given as follows



From the figure,

$$V_{s(n)} = V_{L(n)} = \omega_n \, L \, I_{o(n)} = 2\pi \, n \, f_1 \, L \, I_{o(n)}.$$

or,

$$I_{o(n)} = \frac{V_{s(n)}}{2\pi \, n \, f_1 \, L} = \frac{k \, V_d}{2\pi \, n \, f_1 \, L}.$$

where n and its corresponding k values are taken from Table 1.
The highest ripple component occurs at m_f, which is 1050 Hz, and the peak value is

$$\hat{I}_{o(m_f)} = \frac{1.006 \, V_d}{2\pi \, m_f \, f_1 \, L} = \frac{1.006 \times 542.12}{2 \times \pi \times 21 \times 50 \times 10 \times 10^{-3}} = 8.27\,\text{A}.$$

The RMS ripple component is

$$I_{o(m_f)} = \frac{\hat{I}_{o(m_f)}}{\sqrt{2}} = 5.85\text{A}.$$

(e) **If a Unipolar modulation is used, determine the RMS and frequency of the highest output ripple current component.**
In Unipolar modulation, the highest ripple components occur at $2m_f \pm 1$, which is 2050 Hz or 2150 Hz, and the peak values are

$$\hat{I}_{o(2m_f+1)} = \frac{0.37 \, V_d}{2\pi \, (2m_f - 1) \, f_1 \, L} = \frac{0.37 \times 542.12}{2 \times \pi \times (2 \times 21 - 1) \times 50 \times 10 \times 10^{-3}} = 1.56\,\text{A}.$$
$$\hat{I}_{o(2m_f-1)} = \frac{0.37 \, V_d}{2\pi \, (2m_f + 1) \, f_1 \, L} = \frac{0.37 \times 542.12}{2 \times \pi \times (2 \times 21 + 1) \times 50 \times 10 \times 10^{-3}} = 1.49\,\text{A}.$$

In Unipolar modulation, the highest ripple components occur at $2m_f - 1$, i.e., 2050 Hz. the RMS value is

$$I_{o(2m_f-1)} = \frac{\hat{I}_{o(2m_f-1)}}{\sqrt{2}} = 1.1\text{A}.$$