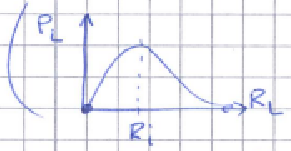
Lektion 2

2.1

$$P = R \cdot I^2, \quad U = (R_i + R_L) I \Rightarrow P_L = R_L \left(\frac{U}{R_i + R_L} \right)^2$$



$R_L = 0 \rightarrow$ Kortsluting, $P_L = 0$

$R_L \rightarrow \infty$, $I \rightarrow 0$, $P_L = 0$

$$\frac{dP_L}{dR_L} = \left[\frac{d}{dx} \frac{g(x)}{h(x)} = \frac{g'(x)h(x) - g(x)h'(x)}{h(x)^2} \right]$$

$$= \frac{U^2 (R_i + R_L) - R_L U^2 2}{(R_i + R_L)^3}$$

$$= U^2 (R_i - R_L) / (R_i + R_L)^3$$

Hitta max: $\frac{dP_L}{dR_L} = 0$ när $R_L = R_i$

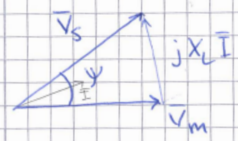
Dubbelkolla maximipunkten mha andraderivatan $\frac{2U^2(R_L - 2R_i)}{(R_L + R_i)^4}$

a)

b) $\eta = \frac{P_L}{P_U} = \frac{R_L I^2}{(R_i + R_L) I^2} = \frac{R_L}{(R_i + R_L)}$

c) Maximal effekt fås när $R_L = R_i$. Sätt in det i uttrycket för effektivitet-> $\eta = \frac{R_i}{R_i + R_i} = 0.5$

2.2



$$\left(\begin{array}{l}\bar{V}_s = \bar{V}_R + jX_L\bar{I} \\ \bar{I} = \dfrac{\bar{V}_s - \bar{V}_R}{jX_L}\end{array}\right)$$

$$\bar{S}_s = 3\cdot\frac{\bar{V}_s}{\sqrt{3}}\bar{I}^* = 3\frac{\bar{V}_s}{\sqrt{3}}\left(\frac{\bar{V}_s - \bar{V}_R}{\sqrt{3}\,j\,X_L}\right)^*$$

$$= j\,3\frac{\bar{V}_s}{\sqrt{3}}\left(\frac{\bar{V}_s - \bar{V}_R}{\sqrt{3}\,X_L}\right)^* = j\frac{\bar{V}_s\bar{V}_s^*}{X_L} - j\frac{\bar{V}_s\bar{V}_R^*}{X_L}$$

$$= j\frac{V_s^2}{X_L} - j\frac{V_sV_R}{X_L}(\cos\psi - j\sin\psi)$$

$$= j\frac{V_s^2}{X_L} - \frac{V_sV_R}{X_L}(j\cos\psi + \sin\psi)$$

$$\bar{S}_s = P_s + jQ_s$$

$$P_s = -\frac{V_sV_R}{X_L}\sin\psi$$

$$Q_s = \frac{V_s^2 - V_sV_R\cos\psi}{X_L}$$

$$\bar{S}_R = 3\frac{\bar{V}_R}{\sqrt{3}}\left(\frac{\bar{V}_s - \bar{V}_R}{\sqrt{3}\,j\,X_L}\right)^*$$

$$= \frac{V_RV_s}{X_L}(j\cos\psi + \sin\psi) - j\frac{V_R^2}{X_L}$$

$$P_R = \frac{V_RV_s}{X_L}\sin\psi$$

$$Q_R = \frac{V_sV_R\cos\psi - V_R^2}{X_L}$$

2.5

120:460 V

$Z_{eq} = 0.018 + j0.042$ p.u.

$S_{base} = 15$ kVA

$V_{baseL} = 120$ V, $V_{baseH} = 460$ V

$(Z_{base} = V_{base}^2 / S_{base})$

$Z_{baseL} = 120^2/15000 = 0.96\ \Omega$

$Z_{baseH} = 460^2/15000 = 14.1\ \Omega$

$(Z_{p.u.} = Z_{actual\ value} / Z_{base} \Rightarrow)$

$Z_{eqL} = Z_{eq}\cdot Z_{baseL} = 0.017 + j0.040\ \Omega$

$Z_{eqH} = Z_{eq}\cdot Z_{baseH} = 0.25 + j0.60\ \Omega$

2.6

First the base values in each zone are determined. $S_{base} = 30$ kVA is the same for the entire network. Also, $V_{base1} = 240$ volts, as specified for zone 1. When moving across a transformer, the voltage base is changed in proportion to the transformer voltage ratings. Thus,

$$V_{base2} = \left(\frac{480}{240}\right)(240) = 480 \quad \text{volts}$$

and

$$V_{base3} = \left(\frac{115}{460}\right)(480) = 120 \quad \text{volts}$$

The base impedances in zones 2 and 3 are

$$Z_{base2} = \frac{V_{base2}^2}{S_{base}} = \frac{480^2}{30{,}000} = 7.86 \quad \Omega$$

and

$$Z_{base3} = \frac{V_{base3}^2}{S_{base}} = \frac{120^2}{30{,}000} = 0.48 \quad \Omega$$

and the base current in zone 3 is

$$I_{base3} = \frac{S_{base}}{V_{base3}} = \frac{30{,}000}{120} = 250 \quad \text{A}$$

Next, the per-unit circuit impedances are calculated using the system base values. Since $S_{base} = 30$ kVA is the same as the kVA rating of transformer T_1, and $V_{base1} =$ 240 volts is the same as the voltage rating of the zone 1 side of transformer T_1, the per-unit leakage reactance of T_1 is the same as its nameplate value, $X_{T1p.u.} = 0.1$ per unit. However, the per-unit leakage reactance of transformer T_2 must be converted from its nameplate rating to the system base. Using (3.3.11) and $V_{base2} = 480$ volts,

$$X_{T2p.u.} = (0.10)\left(\frac{460}{480}\right)^2\left(\frac{30{,}000}{20{,}000}\right) = 0.1378 \ \text{per unit}$$

Alternatively, using $V_{base3} = 120$ volts,

$$X_{T2p.u.} = (0.10)\left(\frac{115}{120}\right)^2\left(\frac{30{,}000}{20{,}000}\right) = 0.1378 \ \text{per unit}$$

which gives the same result. The line, which is located in zone 2, has a per-unit reactance

$$X_{linep.u.} = \frac{X_{line}}{Z_{base2}} = \frac{2}{7.68} = 0.2604 \quad \text{per unit}$$

and the load, which is located in zone 3, has a per-unit impedance

$$Z_{\text{loadp.u.}} = \frac{Z_{\text{load}}}{Z_{\text{base3}}} = \frac{0.9 + j0.2}{0.48} = 1.875 + j0.4167 \quad \text{per unit}$$

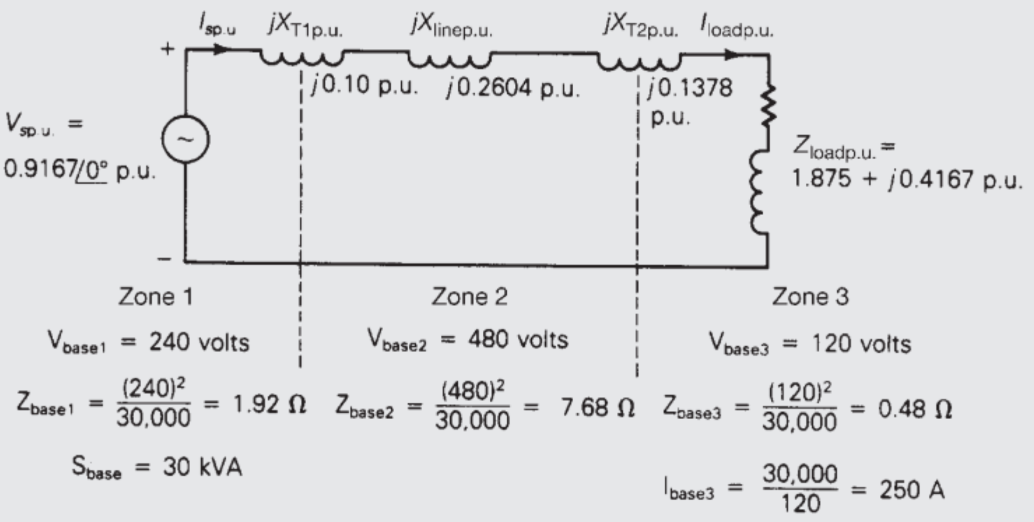
The per-unit circuit is shown in Figure 3.10(b), where the base values for each zone, per-unit impedances, and the per-unit source voltage are shown. The per-unit load current is then easily calculated from Figure 3.10(b) as follows:

$$\begin{aligned}
I_{\text{loadp.u.}} = I_{\text{sp.u.}} &= \frac{V_{\text{sp.u.}}}{j(X_{\text{T1p.u.}} + X_{\text{linep.u.}} + X_{\text{T2p.u.}}) + Z_{\text{loadp.u.}}} \\
&= \frac{0.9167\underline{/0^\circ}}{j(0.10 + 0.2604 + 0.1378) + (1.875 + j0.4167)} \\
&= \frac{0.9167\underline{/0^\circ}}{1.875 + j0.9149} = \frac{0.9167\underline{/0^\circ}}{2.086\underline{/26.01^\circ}} \\
&= 0.4395\underline{/-26.01^\circ} \quad \text{per unit}
\end{aligned}$$

The actual load current is

$$I_{\text{load}} = (I_{\text{loadp.u.}})\mathrm{I}_{\text{base3}} = (0.4395\underline{/-26.01^\circ})\,(250) = 109.9\underline{/-26.01^\circ} \quad \text{A}$$

Note that the per-unit equivalent circuit of Figure 3.10(b) is relatively easy to analyze, since ideal transformer windings have been eliminated by proper selection of base values.



(b) Per-unit circuit