Exercise 1

For a cascade of *n* noisy stages, Friis' formula determines the overall noise factor as

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \ldots + \frac{F_n - 1}{G_1 G_2 \ldots G_{n-1}}$$

where F_i and G_i are the noise factor and power gain of stage *i* in the cascade, respectively. Now consider a *short-range* wireless system consisting of two stages with $G_1G_2 = 100$ (40 dB). G_1 and G_2 should be chosen so that the total power consumption is minimized. Assume the system is bandwidth limited with $P_i = \gamma G_i f$, where γ is a constant and f is the signal frequency.

a) What F would yield minimal power dissipation if $F_1 = 2$ and $F_2 = 10$?

b) How large noise figure can be tolerated for the individual stages if F_1 and F_2 are selected equal for both stages and the power dissipation is minimized?

Exercise 2

The voltage gain of the RF amplifier shown in Figure 1 is $A = g_m Z_0$ at resonance, where $Z_0 = Q\omega_0 L = Q/(\omega_0 C)$. The current through the MOSFET can be described by $I = g_m V_{eff}/2$. How much power can be saved in the circuit if the Q value is increased with a factor of k?

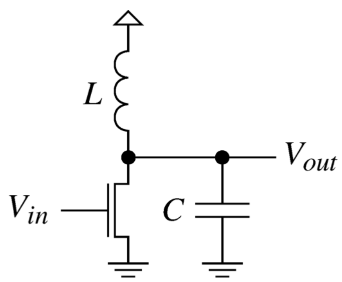

Figure 1. Simple RF amplifier.

Exercise 3

Consider a Low-Noise Amplifier (LNA).

a) How can the linearity be increased?

b) How can the noise be decreased?

c) How does the scaling towards shorter channel length affect Noise Figure (NF) and power gain?

d) How does scaling of power supply voltage affect SNR?

Exercise 4

Consider power amplifier design. How is power consumption traded with linearity?

Exercise 5

The common-gate low-noise amplifier to the left in Figure 2 should be designed with a higher input impedance $Z = Z_1$ than the required Z_0 to save power. The input is then transformed to Z_0 with the addition of the LC-net to the right.

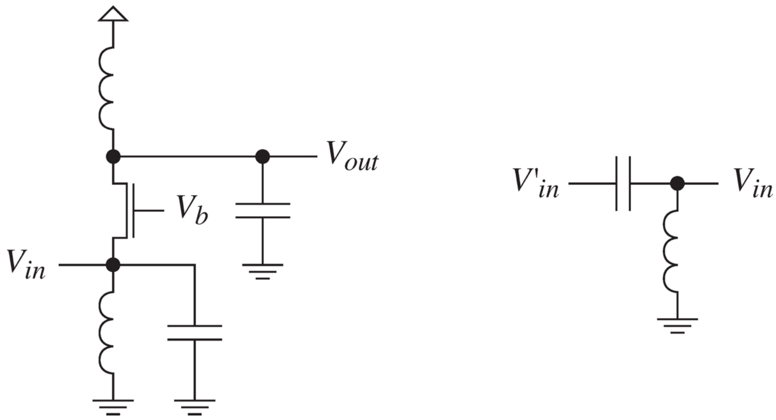

Figure 2. Common-gate low-noise amplifier.

The current through the MOSFET is $I = g_m V_{eff}/2 = V_{eff}/(2Z)$. To maintain a constant IIP3 (linearity) V_b needs to be adjusted so that $V_{eff}' = V_{eff}\sqrt{Z_1/Z_0}$. How much power is saved by the transformation, assuming that the scaling is kept within practical limits?

Exercise 6

a) There is a need of large-value, high-quality inductors in RF circuits. Why is it more power efficient to use an on-chip inductor compared with an external inductor?

b) How may technology scaling benefit low-power operation of a radio system?

c) Why is the conventional superheterodyne architecture avoided in power efficient transceivers?

Exercise 7

The total noise factor F of an RF system with n cascaded components can be calculated from the noise factors F_i and the gains G_i of the individual components i using

Friis' formula, i.e., $F - 1 = \sum_{i=1}^{n} \frac{F_i - 1}{\prod_{j=1}^{i-1} G_j}$. The total gain is given by $G = \prod_{j=1}^{n} G_j$.

Consider the system with two cascaded components depicted in Figure 3. In this exercise it is assumed to be used over *long range*. Hence the design will be limited by both the linearity and the sensitivity.

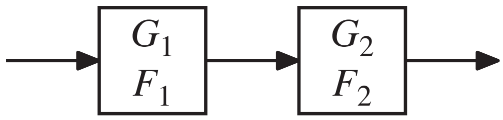

Figure 3. Two cascaded RF components.

The minimal power consumption of this system is given by the formula

$$P_{min} = \frac{IP3}{F - F_1}[\kappa_1(F_2 - 1) + 2\sqrt{\kappa_1\kappa_2 G(F - F_1)(F_2 - 1)} + \kappa_2 G(F - F_1)]$$

where $IP3$ is the third-order input intercept point of the total system, and κ_1, κ_2 are the power linearity parameters of the individual components. Assume that $IP3$ and κ_1, κ_2 are fixed, positive constants in our system of interest.

a) Show that it is sufficient to know the gain of the individual components to determine the minimal power consumption.

b) Assume that we would like to design the system with $\kappa_1 = \kappa_2$, $G_1 \geq 1$, $G_2 \geq 1$, and $G = G_f > 1$, where G_f is a fixed total gain. How should the gain of the individual components be selected to yield the lowest power consumption?

Solution 1

a) $F = F_1 + \dfrac{F_2 - 1}{G_1} \Leftrightarrow G_1 = \dfrac{F_2 - 1}{F - F_1}$

$$P = P_1 + P_2 = \gamma G_1 f + \gamma G_2 f = \gamma f\left(G_1 + \frac{100}{G_1}\right) = \gamma f\left(\frac{F_2 - 1}{F - F_1} + 100\frac{F - F_1}{F_2 - 1}\right)$$

Let $A = \dfrac{P}{\gamma f}$, $B = F - F_1$ and use numerical values $\Rightarrow A = \dfrac{9}{B} + 100\dfrac{B}{9}$

Look for extreme points $\dfrac{dA}{dB} = -\dfrac{9}{B^2} + \dfrac{100}{9} = 0 \Rightarrow B = \pm\dfrac{9}{10}$

Since $A > 0$, $B = 0.9 \Rightarrow F = F_1 + B = 2.9$ (show that this is a minimum…)

b) Using G_2=100/G_1, we obtain $P_{tot} = \gamma f\left(G_1 + \dfrac{100}{G_1}\right)$

Search for extreme points by $\dfrac{dP_{tot}}{dG_1} = \gamma f - \dfrac{100\gamma f}{G_1^2} = 0 \Rightarrow G_1 = \pm 10$

Since $G_1 > 0$, $G_1 = 10$ should be the correct solution

2nd derivative test $\dfrac{d^2 P_{tot}}{\left(dG_1\right)^2} = \dfrac{200\gamma f}{G_1^3} > 0$ for $G_1 = 10 \Rightarrow G_1 = 10$ is a local minimum

Hence $G_1 = G_2 = 10$ minimizes the power dissipation

Use $x = F_1 = F_2$ in Friis' formula for $n = 2$:

$\mathrm{NF} = \mathrm{SNR}_{in} - \mathrm{SNR}_{out} = 10 \log_{10} F \Rightarrow \mathrm{NF} \leq 10 \log_{10} x \approx 4.5$ dB

Solution 2

$$g_m = \frac{2I}{V_{eff}} \Rightarrow A = \frac{2I}{V_{eff}}Z_0 = \frac{2I}{V_{eff}}Q\omega_0 L$$

To maintain the same gain A of the amplifier when Q is increased with a factor of k, the new current is set to $I_{new} = I/k$.

$P = IV_{DD} \Rightarrow P_{new} = I_{new}V_{DD} = IV_{DD}/k = P/k$

Hence $1-1/k$ of the original power consumption can be saved

Solution 3

(solution is missing)

Solution 4

(solution is missing)

Solution 5

$$I' = \frac{1}{2}\frac{V_{eff}'}{Z_1} = \frac{1}{2}\frac{V_{eff}}{Z_1}\sqrt{\frac{Z_1}{Z_0}} = I\sqrt{\frac{Z_0}{Z_1}} \Rightarrow \frac{P'}{P} = \frac{I'}{I} = \sqrt{\frac{Z_0}{Z_1}}$$

Solution 6

a) There is no power wasted in driving the extra stray capacitance of the pad, package and board.

b) The digital parts may have a substantial reduction in power dissipation while the analog part is little affected from scaling down a technology.

c) It has more power consuming stages than a homodyne (zero IF) transceiver or a transceiver that processes the signals with low IF.

Solution 7

a) Noise factor for $n = 2$: $F - 1 = \sum_{i=1}^{2} \frac{F_i - 1}{\prod_{j=1}^{i-1} G_j} = F_1 + \frac{F_2 - 1}{G_1} \Rightarrow F - F_1 = \frac{F_2 - 1}{G_1}$

Total gain for $n = 2$: $G = \prod_{j=1}^{2} G_j = G_1 G_2$

Insert $(F - F_1)$ and G into formula for P_{min}

$$P_{min} = \frac{IP3}{(F - F_1)}[\kappa_1(F_2 - 1) + 2\sqrt{\kappa_1\kappa_2 G(F - F_1)(F_2 - 1)} + \kappa_2 G(F - F_1)]$$

$$P_{min} = \frac{G_1 IP3}{F_2 - 1}[\kappa_1(F_2 - 1) + 2\sqrt{\kappa_1\kappa_2 G_2(F_2 - 1)^2} + \kappa_2 G_2(F_2 - 1)]$$

$$F_2 > 1 \Rightarrow P_{min} = G_1 IP3(\kappa_1 + 2\sqrt{\kappa_1\kappa_2 G_2} + \kappa_2 G_2)$$

Hence, it is sufficient to know the gains G_1 and G_2 to calculate P_{min}

b) $\kappa_2 = \kappa_1 > 0 \Rightarrow P_{min} = \kappa_1 IP3\, G_1(1 + 2\sqrt{G_2} + G_2) = \kappa_1 IP3\, G_1(1 + \sqrt{G_2})^2$

With $G_1 = \frac{G_f}{G_2}$, $P_{min} = \kappa_1 IP3\, G_f\left(\frac{1}{\sqrt{G_2}} + 1\right)^2$, i.e. the higher G_2, the lower P_{min}

The highest G_2 is obtained for the lowest $G_1 = 1$, yielding $G_2 = G_f$