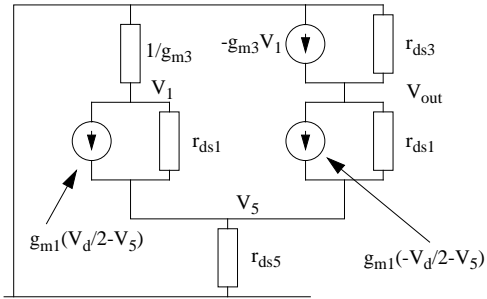


THE DIFFERENTIAL AMPLIFIER

1 Gain

($M_1=M_2$ and $M_3=M_4$) $\Rightarrow r_{ds1} = r_{ds2}$, $g_{m1} = g_{m2}$, $r_{ds3} = r_{ds4}$ and $g_{m3} = g_{m4}$.

The gain is calculated when the output current is 0 (no additional load).



$$\begin{cases} V_1 g_{m3} + g_{m1} \left(\frac{V_d}{2} - V_5 \right) = \frac{V_5 - V_1}{r_{ds1}} \\ g_{m1} \left(\frac{V_d}{2} - V_5 \right) + \frac{V_1 - V_5}{r_{ds1}} + g_{m1} \left(-\frac{V_d}{2} - V_5 \right) + \frac{V_{out} - V_5}{r_{ds1}} = \frac{V_5}{r_{ds5}} \\ g_{m3} V_1 + \frac{V_{out}}{r_{ds3}} + g_{m1} \left(-\frac{V_d}{2} - V_5 \right) + \frac{V_{out} - V_5}{r_{ds1}} = 0 \end{cases}$$

$$V_{out} = \frac{g_{m1} r_{ds1} r_{ds3} (1 + 2g_{m3} r_{ds1} + 4g_{m3} r_{ds5} + 4g_{m1} g_{m3} r_{ds1} r_{ds5}) V_d}{2(r_{ds1} + r_{ds5} + g_{m3} r_{ds1}^2 + r_{ds3} + g_{m3} r_{ds1} r_{ds3} + r_{ds5} (g_{m1} r_{ds1} + 2g_{m3} (1 + g_{m1} r_{ds1}) (r_{ds1} + r_{ds3})))}$$

Assume $g_m \gg g_{ds}$:

$$V_{out} = \frac{r_{ds1} r_{ds3} g_{m1} V_d}{(r_{ds1} + r_{ds3})} = g_{m1} (r_{ds1} \parallel r_{ds3}) V_d$$

2 Output Resistance

The output resistance is calculated when the input signal is 0.

$$\begin{cases} V_1 g_{m3} + g_{m1} \left(\frac{0}{2} - V_5 \right) = \frac{V_5 - V_1}{r_{ds1}} \\ g_{m1} \left(\frac{0}{2} - V_5 \right) + \frac{V_1 - V_5}{r_{ds1}} + g_{m1} \left(-\frac{0}{2} - V_5 \right) + \frac{V_{out} - V_5}{r_{ds1}} = \frac{V_5}{r_{ds5}} \\ g_{m3} V_1 + \frac{V_{out}}{r_{ds3}} + g_{m1} \left(-\frac{0}{2} - V_5 \right) + \frac{V_{out} - V_5}{r_{ds1}} = I_{out} \end{cases}$$

$$\frac{V_{out}}{I_{out}} = \frac{r_{ds3} (r_{ds1} (1 + g_{m3} r_{ds1}) + r_{ds5} (1 + g_{m1} r_{ds1}) (1 + 2g_{m3} r_{ds1}))}{r_{ds1} (1 + g_{m3} r_{ds1}) + r_{ds3} (1 + g_{m3} r_{ds1}) + r_{ds5} (1 + g_{m1} r_{ds1}) (1 + 2g_{m3} (r_{ds1} + r_{ds3}))}$$

Assuming $g_m \gg g_{ds}$

$$\frac{V_{out}}{I_{out}} = \frac{r_{ds3} r_{ds1}}{(r_{ds1} + r_{ds3})} = r_{ds1} \parallel r_{ds3}$$

3 Transconductance

The transconductance is calculated when the output voltage is 0.

$$\begin{cases} V_1 g_{m3} + g_{m1} \left(\frac{V_d}{2} - V_5 \right) = \frac{V_5 - V_1}{r_{ds1}} \\ g_{m1} \left(\frac{V_d}{2} - V_5 \right) + \frac{V_1 - V_5}{r_{ds1}} + g_{m1} \left(-\frac{V_d}{2} - V_5 \right) + \frac{0 - V_5}{r_{ds1}} = \frac{V_5}{r_{ds5}} \\ g_{m3} V_1 + \frac{0}{r_{ds3}} + g_{m1} \left(-\frac{V_d}{2} - V_5 \right) + \frac{0 - V_5}{r_{ds1}} = I_{out} \end{cases}$$

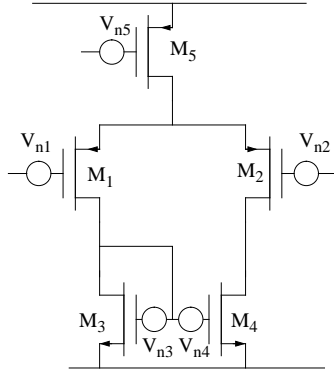
$$\frac{I_{out}}{V_d} = \frac{r_{ds1} g_{m1} (1 + 2g_{m3} (r_{ds1} + 2(1 + g_{m1} r_{ds1}) r_{ds5}))}{2(r_{ds1} (1 + 3g_{m1} r_{ds1}) + (1 + g_{m1} r_{ds1}) (1 + 2g_{m3} r_{ds1}) r_{ds5})}$$

Assuming $g_m \gg g_{ds}$

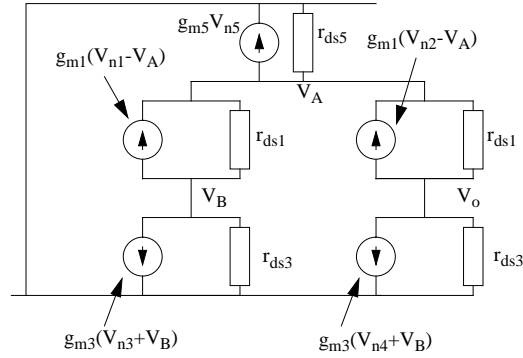
$$\frac{I_{out}}{V_d} = g_{m1}$$

4 Noise

$(M_1=M_2 \text{ and } M_3=M_4) \Rightarrow r_{ds1} = r_{ds2}, g_{m1} = g_{m2}, r_{ds3} = r_{ds4} \text{ and } g_{m3} = g_{m4}$.



Small-Signal Model:



Nodal Equations:

$$\begin{cases} g_{m5}V_{n5} + V_A g_{ds5} = g_{m1}(V_{n1} - V_A) + g_{m1}(V_{n2} - V_A) + (V_B - V_A)g_{ds1} + (V_o - V_A)g_{ds1} \\ g_{m1}(V_{n1} - V_A) + (V_B - V_A)g_{ds1} + g_{m3}(V_{n3} - V_B) + V_B g_{ds3} = 0 \\ g_{m1}(V_{n2} - V_A) + (V_o - V_A)g_{ds1} + g_{m3}(V_{n3} - V_o) + V_o g_{ds3} = 0 \end{cases}$$

Derive the transfer functions for the noise sources:

1. From V_{n1} to V_o : (Set all other sources to 0)

$$\frac{V_o}{V_{n1}} = \frac{g_{m1}(g_{ds1}g_{ds3} + g_{ds3}g_{m1} + 2g_{ds1}g_{m3} + g_{ds5}g_{m3} + 2g_{m1}g_{m3})}{(g_{ds1} + g_{ds3})(g_{ds1}(2g_{ds3} + g_{ds5}) + g_{ds3}(g_{ds5} + 2g_{m1})) + (2g_{ds1} + g_{ds5} + 2g_{m1})g_{m3}}$$

Assuming $g_m \gg g_{ds}$: $V_o/V_{n1} \approx g_{m1}(r_{ds1} \parallel r_{ds3})$

2. From V_{n2} to V_o : (Set all other sources to 0)

$$\frac{V_o}{V_{n2}} = -\frac{V_o}{V_{n1}} \approx -g_{m1}(r_{ds1} \parallel r_{ds3})$$

3. From V_{n3} to V_o : (Set all other sources to 0):

$$\frac{V_o}{V_{n3}} = \frac{g_{m3}(-g_{ds1}(g_{ds1} + g_{m1}) + (2g_{ds1} + g_{ds5} + 2g_{m1})g_{m3})}{(g_{ds1} + g_{ds3})(g_{ds1}(2g_{ds3} + g_{ds5}) + g_{ds3}(g_{ds5} + 2g_{m1})) + (2g_{ds1} + g_{ds5} + 2g_{m1})g_{m3}}$$

Assuming $g_m \gg g_{ds}$: $V_o/V_{n3} \approx g_{m3}(r_{ds1} \parallel r_{ds3})$

4. From V_{n4} to V_o : (Set all other sources to 0):

$$\frac{V_o}{V_{n4}} = \frac{-g_{m3}((g_{ds1} + g_{ds5})(g_{ds1} + g_{m1}) + (g_{ds1} + 2g_{ds3})g_{m1} + (2g_{ds1} + g_{ds5} + 2g_{m1})g_{m3})}{(g_{ds1} + g_{ds3})(g_{ds1}(2g_{ds3} + g_{ds5}) + g_{ds3}(g_{ds5} + 2g_{m1})) + (2g_{ds1} + g_{ds5} + 2g_{m1})g_{m3}}$$

Assuming $g_m \gg g_{ds}$: $V_o/V_{n4} \approx -g_{m3}(r_{ds1} \parallel r_{ds3})$

5. From V_{n5} to V_o : (Set all other sources to 0):

$$\frac{V_o}{V_{n5}} = \frac{-(g_{ds1} + g_{m1})g_{m5}}{(g_{ds1}(2g_{ds3} + g_{ds5}) + g_{ds3}(g_{ds5} + 2g_{m1})) + (2g_{ds1} + g_{ds5} + 2g_{m1})g_{m3}}$$

Assuming $g_m \gg g_{ds}$: $V_o/V_{n5} \approx \frac{-g_{m5}}{g_{m3}}$

The total noise spectral density at the output is given by the sum of the spectral densities from each noise source multiplied by the transfer function squared.