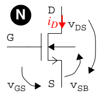
Lecture 2

Transistor models

• Regions of operation: NMOS



λ in the below equations is often neglected.

Cut Off : $V_{GS} < V_T$

$$I_D = 0$$

Triode (Linear) Region : $0 < V_{GS} - V_T > V_{DS}$

$$I_D = \beta((V_{GS} - V_T) - V_{DS}/2)V_{DS}(1 + \lambda V_{DS})$$

Active (Saturation) Region: $0 < V_{GS} - V_T < V_{DS}$

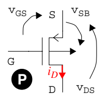
$$I_D = (\beta/2) \cdot (V_{GS} - V_T)^2(1 + \lambda V_{DS})$$

$$\beta = K' \cdot W/L = \mu_0 C_{ox}\frac{W}{L}$$

Bulk connection (Body Effect)

$$V_T = V_{T0} + \gamma(\sqrt{2|\phi_F| + V_{SB}} - \sqrt{2|\phi_F|})$$

• Regions of operation: PMOS



λ in the below equations is often neglected.

Cut Off : $V_{SG} < |V_T|$

$$I_D = 0$$

Triode (Linear) Region : $0 < V_{SG} - |V_T| > V_{SD}$

$$I_D = \beta((V_{SG} - |V_T|) - V_{SD}/2)V_{SD}(1 + \lambda V_{SD})$$

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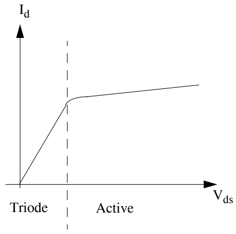
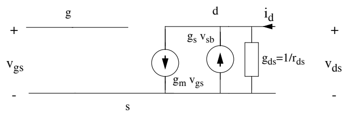


Small Signal Parameters

• Low Frequency



Active Region

$$g_m = \frac{\partial I_d}{\partial V_{gs}} = \mu_0 C_{ox}\frac{W}{L}(V_{gs} - V_T) = \sqrt{2\mu_0 C_{ox}\frac{W}{L}I_d}$$

$$g_{ds} = \frac{\partial I_d}{\partial V_{ds}} = \lambda I_d$$

where $\lambda \propto 1/L$

$$g_s = \frac{\partial I_d}{\partial V_{bs}} = \frac{\gamma g_m}{2\sqrt{V_{sb} + |2\phi_F|}}$$

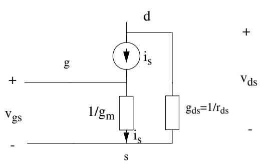
Triode Region

$$g_m = \frac{\partial I_d}{\partial V_{gs}} \approx \beta V_{ds}$$

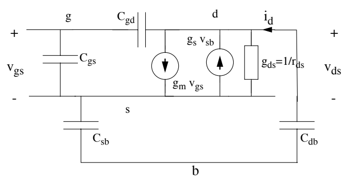
$$g_{ds} = \frac{\partial I_d}{\partial V_{ds}} = \mu_0 C_{ox}\frac{W}{L}(V_{gs} - V_T - V_{ds})$$

$$g_s = \frac{\partial I_d}{\partial V_{bs}} = \frac{\beta\gamma V_{ds}}{2\sqrt{V_{sb} + |2\phi_F|}}$$

• The T model



• Capacitors



Active

$$C_{gs} = \frac{2}{3}WLC_{ox} + (WL_{ov}C_{ox})$$

$$C_{sb} \approx (A_{source} + A_{channel})\frac{C_{j0}}{\sqrt{1 + \frac{V_{BS}}{\Phi_0}}}$$

$$C_{db} \approx (A_{drain})\frac{C_{j0}}{\sqrt{1 + \frac{V_{BS}}{\Phi_0}}}$$

$$C_{gd} = WL_{ov}C_{ox}$$

Triode

$$C_{gs} = C_{gd} = \frac{1}{2}WLC_{ox} + (WL_{ov}C_{ox})$$

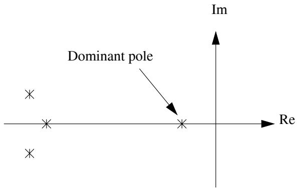
$$C_{sb} \approx \left(A_{source} + \frac{A_{channel}}{2}\right)\frac{C_{j0}}{\sqrt{1 + \frac{V_{BS}}{\Phi_0}}}$$

$$C_{db} \approx \left(A_{drain} + \frac{A_{channel}}{2}\right)\frac{C_{j0}}{\sqrt{1 + \frac{V_{BS}}{\Phi_0}}}$$

Frequency Performance

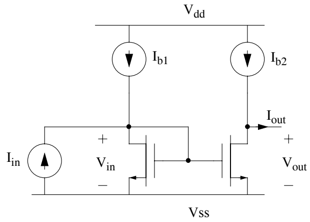
It is common that the transfer function A(s) of analog circuits have one dominant pole and several high frequency poles and zeros.

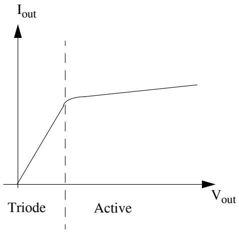
Ex.



If the dominant pole and the high frequency poles are widely separated the performance of the circuit is mainly determined by the dominant pole and for hand calculations we need only consider this pole.

Simple Current Mirrors

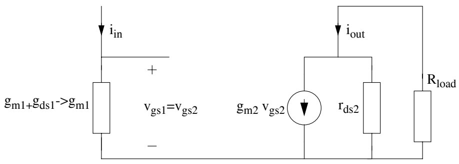




For proper operation:

$V_{in} > V_T$ and $V_{out} > V_{eff} = V_{gs1} - V_T$

- **Small Signal Model**



Low frequency transfer function

$$\frac{i_{out}}{i_{in}} = \frac{g_{m2}}{g_{m1}} \cdot \frac{r_{ds2}}{r_{ds2} + R_{load}} \cdot i_{in}$$

Input and Output Impedance

Thus r_{ds2} should be large and R_{load} small.

$r_{in} \approx 1/g_{m1}$ and $r_{out} = r_{ds2}$

Large $g_{m1} = \sqrt{2K'(W/L)_1 I_{b1}}$ => Large $(W/L)_1$ and I_{b1}

Large $r_{ds2} = \frac{1}{\lambda I_{b2}} \propto \frac{L}{I_{b2}}$ => Large L and small I_{b2}

Current Gain

Ideal Current gain ($R_{load} = 0$)

$$\frac{i_{out}}{i_{in}} = \frac{g_{m2}}{g_{m1}} = \sqrt{\frac{(W/L)_2 I_{b2}}{(W/L)_1 I_{b1}}}$$

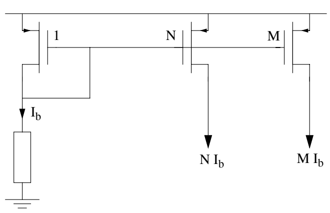
To simplify layout: if we need the current gain G_c, choose $\frac{(W/L)_2}{(W/L)_1} = G_c$ and $\frac{I_{b2}}{I_{b1}} = G_c$

Dominant Pole

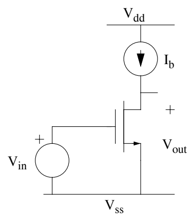
Assume for simplicity that both transistors are equal

$$|p_1| = \frac{g_{m1}}{C_{gs1} + C_{gs2}} = \frac{\sqrt{2K'(W/L)I_b}}{C_{ox}WL + C_{ox}WL} \propto \frac{\sqrt{I_b}}{W^{0.5}L^{1.5}}$$

Bias Circuit



Common Source Amplifier

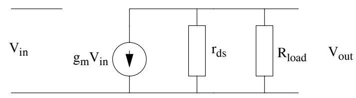


For proper operation :

$V_{in} > V_T$ and

$V_{out} > V_{eff} = V_{gs1} - V_T$

Small Signal Model



Gain

Low frequency gain

$$A = v_{out}/v_{in} = -g_m \cdot (r_{ds} \| R_{load})$$

R_{load} should be large =>

$$A \approx -\sqrt{2K'(W/L)I_b} \cdot \frac{1}{\lambda I_b} \propto \sqrt{\frac{WL}{I_b}}$$

Input and Output Impedance

$r_{in} \approx \infty$ and $r_{out} = r_{ds} \| R_{load}$

Dominant Pole

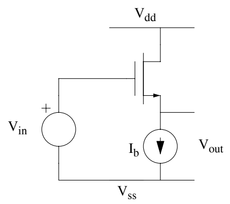
$$|p| = \frac{1}{R_{source}(C_{gs} + C_{gd}(1 + A)) + r_{ds}(C_{gd} + C_{load})}$$

usually the first term in the denominator dominates:

$$|p| = \frac{1}{R_{source}(C_{gs} + C_{gd}(1 + A))}$$

C_{gs} is in the expression increased by the factor $A + 1$ where A is the DC-gain of the amplifier. This effect is refered to as the **Miller effect**.

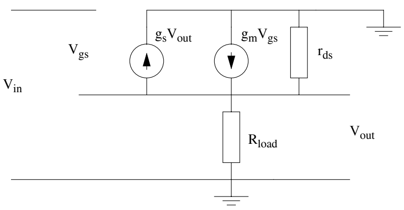
Common Drain Amplifier (Source Follower)



For proper operation :

$$V_{in} - V_{out} > V_T$$

Small Signal Model



Gain

Low frequency gain

$$A = \frac{v_{out}}{v_{in}} = \frac{g_m}{g_m + g_s + g_{ds} + 1/R_{load}} \approx 1$$

$g_s \approx g_m/5$

Input/Output Inpedance

$$r_{in} = \infty \text{ and } r_{out} \approx \frac{1}{g_m}$$

Dominant Pole

$$|p| \approx \frac{g_m}{C_{gs} + C_{load}}$$

Zero:

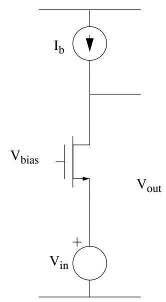
$$z = -\frac{g_m}{C_{gs}}$$

This zero is usually located at a higher frequency than the dominant pole when taking all parasitic capacitors into account.

Note : When taking input and output capacitors into account there will be two poles in the transfer function that for certain parameter values will give complex poles => non-monotonic settling. (p. 158-60 in the book).

The advantages of the source follower are the high bandwidth and the low output impedance. Therefore it is commonly used as a buffer stage or output stage of amplifiers. A low output impedance is desirable since the load will then have a small influence on the gain and bandwidth of the amplifier.

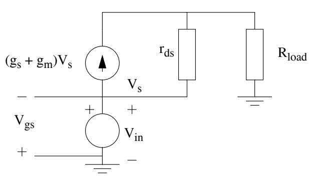
Common-Gate Amplifier



For proper operation :

$$V_{bias} - V_{in} > V_T \text{ and } V_{out} > V_{bias} - V_T$$

Small Signal Model



Gain

$$\frac{v_{out}}{v_{in}} \approx \frac{g_m}{g_{ds} + 1/R_{load}}$$

same as for the Common source amplifier.

Impedance

$$r_{in} = \frac{1}{g_m}\left(1 + \frac{R_{load}}{r_{ds}}\right) \text{ and } r_{out} = r_{ds} \| R_{load}$$

Dominant Pole

$$|p| = \frac{g_m}{C_{gs}} \text{ or } |p| = \frac{1}{r_{ds}C_{load}}$$