Härledning av aktiv effekt

$$p(t) = u(t) \cdot i(t) = \hat{U}\sin(\omega t + \varphi) \cdot \hat{J}\sin(\omega t)$$

DÄR φ ÄR VINKELN MELLAN SPÄNNINGEN u OCH STRÖMMEN i

$$P = \frac{1}{T}\int_0^T p(t)\,dt =$$

$$= \frac{1}{T}\int_0^T \hat{U}\sin(\omega t + \varphi)\,\hat{J}\sin(\omega t)\,dt =$$

$$= \left|\ \omega = 2\pi f = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega}\ \right| =$$

$$= \frac{\hat{U}\hat{J}\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} \left(\cos(\omega t)\sin\varphi + \sin(\omega t)\cos\varphi\right)\sin(\omega t)\,dt =$$

$$= \frac{\hat{U}\hat{J}\omega}{2\pi}\sin\varphi\int_0^{\frac{2\pi}{\omega}} \sin(\omega t)\cos(\omega t)\,dt +$$

$$+ \frac{\hat{U}\hat{J}\omega}{2\pi}\cos\varphi\int_0^{\frac{2\pi}{\omega}} \sin^2(\omega t)\,dt =$$

$$= \left|\ \begin{aligned} \sin(\omega t)\cos(\omega t) &= \frac{\sin(2\omega t)}{2} \\ \sin^2(\omega t) &= \frac{1-\cos(2\omega t)}{2} \end{aligned}\ \right| =$$

$$= \frac{\hat{U}\hat{J}w}{4\pi} \sin\varphi \int_0^{\frac{2\pi}{w}} \sin(2wt)\,dt \; +$$

$$+ \frac{\hat{U}\hat{J}w}{4\pi} \cos\varphi \int_0^{\frac{2\pi}{w}} 1\,dt \; -$$

$$- \frac{\hat{U}\hat{J}w}{4\pi} \cos\varphi \int_0^{\frac{2\pi}{w}} \cos(2wt)\,dt \; =$$

$$= \frac{\hat{U}\hat{J}w}{4\pi} \sin\varphi \left[\frac{-\cos(2wt)}{2w} \right]_0^{\frac{2\pi}{w}} \; +$$

$$+ \frac{\hat{U}\hat{J}w}{4\pi} \cos\varphi \left[t \right]_0^{\frac{2\pi}{w}} \; -$$

$$- \frac{\hat{U}\hat{J}w}{4\pi} \cos\varphi \left[\frac{\sin(2wt)}{2w} \right]_0^{\frac{2\pi}{w}} \; =$$

$$= \frac{\hat{U}\hat{J}w}{4\pi} \sin\varphi \left[-\frac{1}{2w} + \frac{1}{2w} \right] \; +$$

$$+ \frac{\hat{U}\hat{J}w}{4\pi} \cos\varphi \left[\frac{2\pi}{w} - 0 \right] \; -$$

$$- \frac{\hat{U}\hat{J}w}{4\pi} \cos\varphi \left[0 - 0 \right] = \frac{\hat{U}\hat{J}}{2} \cos\varphi$$

$$= \frac{\hat{U}}{\sqrt{2}} \cdot \frac{\hat{J}}{\sqrt{2}} \cdot \cos\varphi = UJ\cos\varphi$$