7.1-4

a) $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_0^T e^{-at} e^{-j\omega t} dt = \int_0^T e^{-(a+j\omega)t} dt$

$$= \left[ \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^T = \frac{1 - e^{-(a+j\omega)T}}{a+j\omega}$$

b) $X(\omega) = \int_0^T e^{at} \cdot e^{-j\omega t} dt = \int_0^T e^{(a-j\omega)t} dt = \frac{1 - e^{(a-j\omega)T}}{-a+j\omega}$

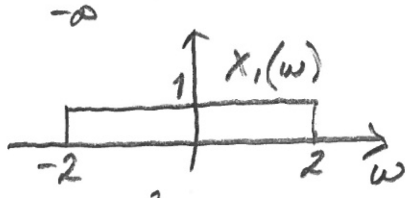
7.1-5

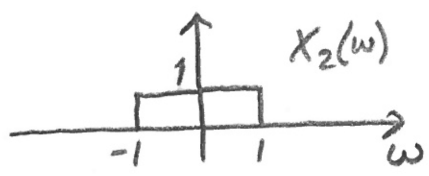
a) $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_0^1 4 \cdot e^{-j\omega t} dt + \int_1^2 2 \cdot e^{-j\omega t} dt$

$$= \frac{4}{-j\omega} \left[ e^{-j\omega t} \right]_0^1 + \frac{2}{-j\omega} \left[ e^{-j\omega t} \right]_1^2 = \frac{4 - 2e^{-j\omega} - 2e^{-j2\omega}}{j\omega}$$

7.1-6

b) $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left( X_1(\omega) + X_2(\omega) \right) e^{j\omega t} d\omega$

där 

& 

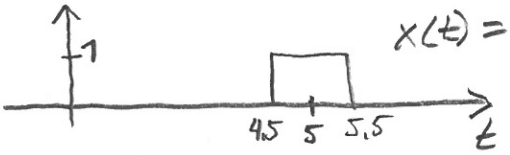

$$\Rightarrow x(t) = \frac{1}{2\pi} \int_{-2}^{2} 1 \cdot e^{j\omega t} d\omega + \frac{1}{2\pi} \int_{-1}^{1} 1 \cdot e^{j\omega t} d\omega =$$

$$= \frac{e^{j2t} - e^{-j2t}}{2j \cdot \pi t} + \frac{e^{jt} - e^{-jt}}{2j \cdot \pi t} = \frac{\sin(2t) + \sin(t)}{\pi t}$$

$$\left( \begin{aligned} &= \frac{2}{\pi} \cdot \mathrm{sinc}(2t) + \frac{1}{\pi} \cdot \mathrm{sinc}(t) \\ &= \frac{2}{\pi} \cdot \mathrm{sinc}_N\left(\frac{2t}{\pi}\right) + \frac{1}{\pi} \cdot \mathrm{sinc}_N\left(\frac{t}{\pi}\right) \end{aligned} \right)$$

Anm: Man kan naturligtvis även beräkna $x(t)$ genom att dela upp inverstransformintegralen i 3 tidsintervall. $\int_{-2}^{-1} + \int_{-1}^{1} + \int_{1}^{2}$

7.2-2

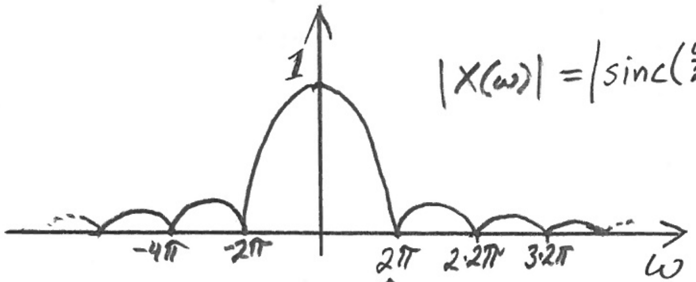



$x(t) = \text{rect}(t-5)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_{4.5}^{5.5} 1 \cdot e^{-j\omega t} dt = \left[ \frac{e^{-j\omega t}}{-j\omega} \right]_{4.5}^{5.5}$$

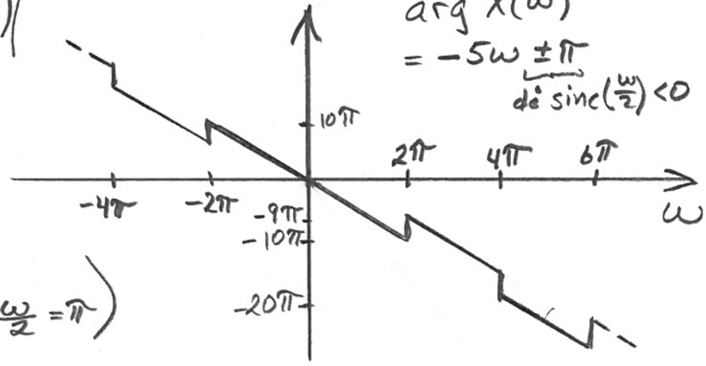
$$= \frac{e^{-j4{,}5\omega} - e^{-j5.5\omega}}{j\omega} = \frac{\left(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}\right) e^{-j5\omega}}{2j \cdot \frac{\omega}{2}}$$

$$= \frac{\sin\left(\frac{\omega}{2}\right)}{\frac{\omega}{2}} \cdot e^{-j5\omega} = \text{sinc}\left(\frac{\omega}{2}\right) \cdot e^{-j5\omega}$$ V.S.V.




$\frac{2\pi}{\text{pulsbredden}}$

$\left( \sin\left(\frac{\omega}{2}\right) = 0 \right.$ första gången, för $\omega > 0$, vid $\left. \frac{\omega}{2} = \pi \right)$




7.2-4

a) $$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega = \frac{1}{2\pi} \int_{-\omega_0}^{\omega_0} 1 \cdot e^{-j\omega t_0} \cdot e^{j\omega t} d\omega$$

$$= \frac{1}{2\pi} \left[ \frac{e^{j\omega(t-t_0)}}{j(t-t_0)} \right]_{-\omega_0}^{\omega_0} = \frac{\sin[\omega_0(t-t_0)]}{\pi(t-t_0)} = \frac{\omega_0}{\pi} \text{sinc}[\omega_0(t-t_0)]$$

$$= \frac{\omega_0}{\pi} \text{sinc}_N \left[ \frac{\omega_0}{\pi}(t-t_0) \right]$$

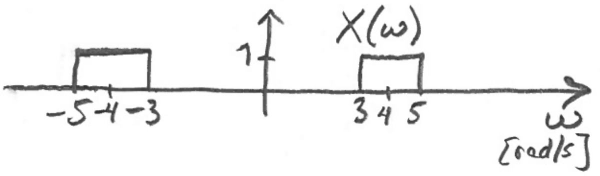
b) $$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega = \frac{1}{2\pi} \left( \int_{-\omega_0}^{0} 1 \cdot \underbrace{e^{j\frac{\pi}{2}}}_{=j} \cdot e^{j\omega t} d\omega + \int_{0}^{\omega_0} 1 \cdot \underbrace{e^{-j\frac{\pi}{2}}}_{=-j} e^{j\omega t} d\omega \right)$$

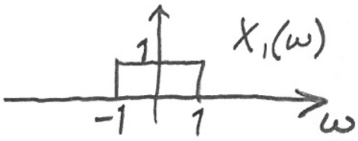
$$= \frac{j}{2\pi} \left( \left[ \frac{e^{j\omega t}}{jt} \right]_{-\omega_0}^{0} - \left[ \frac{e^{j\omega t}}{jt} \right]_{0}^{\omega_0} \right) = \frac{2 - (e^{j\omega_0 t} + e^{-j\omega_0 t})}{2\pi t}$$

$$= \frac{1}{\pi t} \left( 1 - \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \right) = \frac{1 - \cos(\omega_0 t)}{\pi t}$$

7.3-7

a)




$$\Rightarrow X(\omega) = X_1(\omega+4) + X_1(\omega-4)\ , \text{ där } X_1(\omega) = \mathrm{rect}\left(\tfrac{\omega}{2}\right)$$

Tab. 3:13 $\left(\mathrm{sinc}_N(at) = \mathrm{sinc}(a\pi t) \Longleftrightarrow \frac{1}{a}\mathrm{rect}\left(\frac{\omega}{2\pi a}\right)\right)$, med

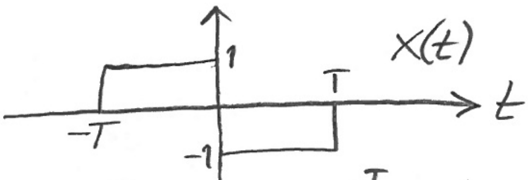
$$X_1(\omega) = \frac{1}{\pi}\cdot\pi\cdot\mathrm{rect}\left(\frac{\omega}{2\pi\cdot\frac{1}{\pi}}\right)\ \left(a=\tfrac{1}{\pi}\right) \Rightarrow \underline{x_1(t) = \frac{1}{\pi}\mathrm{sinc}_N\left(\frac{t}{\pi}\right) = \frac{1}{\pi}\mathrm{sinc}(t)}$$

Tab 2:9, frekvensskift $\left(x(t)e^{j\omega_0 t} \Longleftrightarrow X(\omega-\omega_0)\right)$

$$\Rightarrow x(t) = x_1(t)e^{-j4t} + x_1(t)e^{j4t} = 2\cdot x_1(t)\,\frac{e^{j4t}+e^{-j4t}}{2}$$

$$= 2x_1(t)\cdot\cos(4t) = \frac{2}{\pi}\mathrm{sinc}_N\left(\frac{t}{\pi}\right)\cos(4t)$$

7.3-10



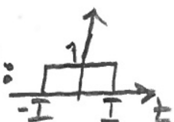

a) $$X(\omega) = \int_{-T}^{0} 1\cdot e^{-j\omega t}dt - \int_{0}^{T} 1\cdot e^{-j\omega t}dt = \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-T}^{0} - \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{0}^{T}$$

$$= \frac{e^0 - e^{j\omega T} - e^{-j\omega T} + e^0}{-j\omega} = \frac{2j(1-\cos(\omega T))}{\omega} = \frac{4j}{\omega}\sin^2\left(\frac{\omega T}{2}\right)$$

b) Tab. 3:12 $\Rightarrow \mathrm{rect}\left(\frac{t}{\tau}\right) \Longleftrightarrow \tau\,\mathrm{sinc}_N\left(\frac{\omega\tau}{2\pi}\right)\ \left[= \tau\cdot\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)\right]$

Tidsskiftning (Tab. 2:8): $x(t-t_0) \Longleftrightarrow X(\omega)e^{-j\omega t_0}$

Här: $x(t) = x_2\left(t+\frac{T}{2}\right) - x_2\left(t-\frac{T}{2}\right)$, där $x_2(t) = \mathrm{rect}\left(\frac{t}{T}\right)$:




$$\Rightarrow X(\omega) = X_2(\omega)\cdot e^{j\omega\frac{T}{2}} - X_2(\omega)\cdot e^{-j\omega\frac{T}{2}} = 2j\,X_2(\omega)\cdot\sin\left(\frac{\omega T}{2}\right)$$

$$= 2j\Big(T\cdot\underbrace{\mathrm{sinc}_N\left(\frac{\omega T}{2\pi}\right)}_{=\sin\left(\frac{\omega T}{2}\right)/\frac{\omega T}{2}}\Big)\cdot\sin\left(\frac{\omega T}{2}\right) = \frac{4j}{\omega}\sin^2\left(\frac{\omega T}{2}\right)$$

c) $$\frac{du(t)}{dt} = \delta(t) \Rightarrow \frac{dx(t)}{dt} = \delta(t+T) - 2\delta(t) + \delta(t-T)$$

$$\Rightarrow \mathcal{F}\left\{\frac{dx(t)}{dt}\right\} = j\omega X(\omega) = 1\cdot e^{j\omega T} - 2 + 1e^{-j\omega T} =$$

$$= -2(1-\cos(\omega T)) = -4\sin^2\left(\frac{\omega T}{2}\right)$$

$$\Rightarrow X(\omega) = \frac{-4}{j\omega}\sin^2\left(\frac{\omega T}{2}\right) = \frac{4j}{\omega}\sin^2\left(\frac{\omega T}{2}\right)$$

7.3-11

a) $X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$

$$\Rightarrow \frac{dX(\omega)}{d\omega} = \int_{-\infty}^{\infty} x(t) \cdot \frac{d}{d\omega} e^{-j\omega t} dt = \int_{-\infty}^{\infty} \underbrace{x(t) \cdot (-jt)}_{=\mathcal{F}^{-1}\left\{\frac{dX(\omega)}{d\omega}\right\}} e^{-j\omega t} dt$$

$$\Rightarrow -jt\,x(t) \Longleftrightarrow \frac{dX(\omega)}{dt}$$

V.S.V.

b) Tab. 3:5 $\Rightarrow$ $e^{-at} \cdot u(t) \Longleftrightarrow \frac{1}{a+j\omega}$

Uppgift a) med $x(t) = e^{-at} \cdot u(t)$ $\Rightarrow$

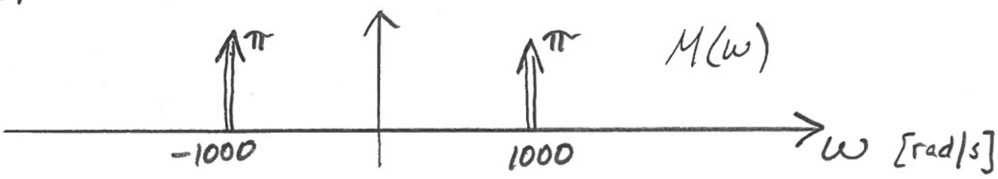
Inre derivatan

$$= -jt \cdot e^{-at} \cdot u(t) \Longleftrightarrow \frac{d}{d\omega}\left\{\frac{1}{a+j\omega}\right\} = \frac{-1 \cdot j}{(a+j\omega)^2}$$

$$\Rightarrow t \cdot e^{-at} \cdot u(t) \Longleftrightarrow \frac{1}{(a+j\omega)^2}$$

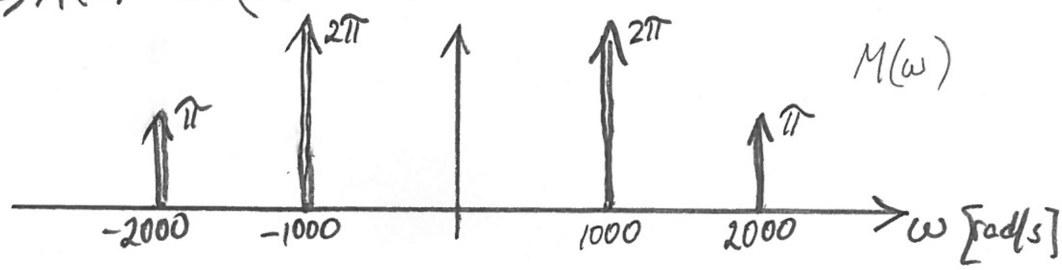
7.7-1 a) Formelsaml. Tab. 3:22 $\Rightarrow$ $\cos(\omega_0 t) \Longleftrightarrow \pi\big(\delta(\omega+\omega_0) + \delta(\omega-\omega_0)\big)$

i) $m(t) = \cos(1000t) \Rightarrow M(\omega) = \pi\big(\delta(\omega+1000) + \delta(\omega-1000)\big)$




ii) $m(t) = 2\cos(1000t) + \cos(2000t)$

$$\Rightarrow M(\omega) = 2\pi\big(\delta(\omega+1000) + \delta(\omega-1000)\big) + \pi\big(\delta(\omega+2000) + \delta(\omega-2000)\big)$$



7.1-5 b) $X(\omega) = \frac{2}{\tau\omega^2}\left(\cos(\omega\tau) + \omega\tau\sin(\omega\tau) - 1\right)$

7.1-8 $X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$ ; $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t} d\omega$

$\Rightarrow X(0) = \int_{-\infty}^{\infty} x(t) dt \quad (\circ)$ ; $\Rightarrow x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) d\omega \quad (\circ\circ)$

Vi har $\operatorname{sinc}(t) \Longleftrightarrow \pi \cdot \operatorname{rect}\left(\frac{\omega}{2}\right)$ (Tab 3:13) $\Rightarrow$

$(\circ) \Rightarrow \pi \cdot \operatorname{rect}(0) = \pi = \int_{-\infty}^{\infty} \operatorname{sinc}(t)\, dt$ V.S.V.

Vi har även $\operatorname{sinc}^2(t) \Longleftrightarrow \pi \cdot \Delta\left(\frac{\omega}{4}\right)$ (Tab. 3:15) $\Rightarrow$

$\overset{(\circ)}{\Rightarrow} \pi \cdot \Delta(0) = \pi = \int_{-\infty}^{\infty} \operatorname{sinc}^2(t)\, dt$ V.S.V.

7.3-1 b) & c) : Använd Tab. 3:22 (Bokens Tab. 7.1:9) i b) och Tab 3:23 i c) (Bokens Tab. 7.1:10)
samt dualitetsegenskapen i Tab. 2:5 (Bokens Tab 7.2)
dvs. om $x(t) \Longleftrightarrow X(\omega)$ så gäller $X(t) \Longleftrightarrow 2\pi x(-\omega)$
$\left( X(t) = X(\omega)\big|_{\omega = t} \qquad x(-\omega) = x(t)\big|_{t=-\omega} \right)$

7.3-3 a) $X(\omega) = \frac{j4}{\omega}\sin^2\left(\frac{\omega T}{2}\right)$ b) $X(\omega) = \frac{1 + e^{-j\pi\omega}}{1-\omega^2}$

c) $X(\omega) = \frac{j\omega + e^{-j\frac{\pi\omega}{2}}}{1-\omega^2}$ d) $X(\omega) = \frac{1 - e^{-(a+j\omega)T}}{a + j\omega}$

7.3-10
c) Se lösning på blad 3, tillsammans med lösningarna till 7.3-10 a) & b)